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Minimum Price Guarantees in a Consumer Search Model

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Minimum Price Guarantees In a Consumer Search Model

Maarten Janssen* Alexei Parakhonyak†

October 12, 2009

Abstract

This paper is the first to examine the effect of minimum price guarantees in a *sequential* search model. Minimum price guarantees are not advertised and only known to consumers when they come to the shop. We show that in such an environment, minimum price guarantees increase the value of buying the good and therefore increase consumers' reservation prices. This increase is so large that even after accounting for the fact that some consumers will buy at lower prices, firms profits are larger under minimum price guarantees than without it. We also show that an equilibrium where all firms offer minimum price guarantees does not exist because of a free-riding problem. Minimum price guarantees can only be an equilibrium phenomenon in an equilibrium where firms randomize their decision to offer these guarantees.

JEL Classification: D40; D83; L13

Keywords: Sequential Search, Minimum Price Guarantees, Welfare Analysis

1 Introduction

It is well known that minimum price guarantees (MPGs) of one sort or the other are found in many sectors and industries. In retail markets, minimum price guarantees (MPGs) often take the form that sellers offer consumers who buy their products to match any other price a competitor charges for identical products provided that they have proof that an identical product is sold by a competitor at a nearby shop within a well-defined time period. It is this type of pricing matching policy that is our main interest in this paper. Major department stores, electronic goods stores and many other retail companies offer MPG in order to insure their potential clients against the possibility that they later regret buying the good if a lower price has been found in a competitor's

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store. Alternative forms of MPGs offer to give back $(100+x)\%$ of the price difference (so called price beating strategies) or offer a ‘free lunch’ in addition to matching prices (see, e.g., IKEA stores).¹ Most firms give the price difference only to consumers who provide evidence of lower prices elsewhere and do not commit to change list prices.²

The effect of MPGs on the (pricing) behavior of competitors has been discussed in the economics as well as in the business and the law literature. The main conclusion from these literatures is that despite the appearance of creating additional competitive pressure on the pricing behaviour of firms, MPGs are in fact highly anticompetitive. One argument that has been made (cf., Salop (1986)) is that MPGs facilitate collusion as they remove the incentives to undercut. MPGs, so it is argued, do not just contain information for consumers, but in fact convey the information to competitors that any attempt to undercut will be automatically followed, i.e., MPGs work as a trigger strategy that helps firms to collude. Moreover, MPGs are an extremely cheap way of doing so as firms do not have to spend any resources on monitoring competitor’s behaviour. Although some MPGs take the form that firms ex ante commit to change their list prices if they are informed that a competitor has a lower price (see above), most MPGs restrict the MPG to the client that has informed the firm of a lower price elsewhere, i.e., list prices are unaffected. This means that most MPGs actually are dissimilar to trigger strategies and it is therefore unclear whether they really support collusive practices.

A second argument that has been made (cf., Png and Hirschleifer (1987)) is that MPGs are an effective way to price discriminate between shoppers and non-shoppers. In the absence of MPGs, the activity of shoppers forces firms to reduce prices market-wide. Shoppers provide a positive externality to non-shoppers and force firms to set more competitive prices. With MPGs, however, the effect of the disciplining power of shoppers is limited to these shoppers themselves according to Png and Hirschleifer and act as a price discrimination mechanism for firms that can set high list prices and provide shoppers with discounts (see, also, Edlin (1997)). One shortcoming of the model proposed by Png and Hirschleifer is that the behavior of shoppers and non-shoppers is exogenously imposed: shoppers always compare all prices, and more importantly, non-shoppers always go to one shop and buy if the price charged is below their willingness to pay. The effect MPGs may have on the search behavior of consumers is not analyzed.

According to both arguments discussed so far, the imposition of MPGs reduces economic welfare and this had led Edlin (1997) to investigate the legal possibilities to prohibit MPGs under the Sherman Act. Recently, Moorthy and Winter (2006) have argued that MPGs may also have a pro-competitive effect in case products are horizontally differentiated and firms have different production

¹The biggest supermarket in the Netherlands, Albert Heijn, introduced in spring 2009 a policy that gave customers a free apple pie in addition to “all your money back policy” in case customers could show that other shops had lower prices for identical products.

²There are, however, some firms that commit to lowering list prices if competitors offer lower prices (see, e.g., Comet Services at comet.co.uk).

costs. In such a context MPGs may signal to consumers that the firm under consideration really has a lower price. The lower price that is charged generates sufficient additional demand to compensate the firm for the lower profit per unit. High cost firms may find it too expensive to imitate the low pricing behavior of low cost firms, thereby allowing MPGs to work as a signalling device. Moorthy and Winter’s model nicely illustrates how MPGs may work in markets with product heterogeneity. Most MPGs clauses, however, stipulate that the guarantee only comes into effect if prices of identical products at nearby shops are compared. This means that Moorthy and Winter’s analysis is restricted to markets where geographical differentiation is important and transportation costs are high. Chen et al. (2001) also show that price matching policies may have pro-competitive effects in case they are pre-announced and there are consumers who prefer to shop at a particular store but are mindful of saving opportunities. In this paper the “search” behaviour of customers is also exogenously given as in Png and Hirschleifer (1987) and Varian (1980).

In this paper, we argue that MPGs have an important effect on the search behaviour of consumers which so far has not neglected in the literature on minimum price guarantees. To study this effect, we cast our model in a conventional sequential search setup a la Stahl (1989). Moreover, we assume that there is a certain probability that after the purchase a customer is informed about another price quotation. This probability represents the level of information communication among the customers (as in Galeotti (2009)). In such a setting, the main decision consumers have to make concerns their reservation price, i.e., the maximal price at which they will buy. We show that an MPG increases this reservation price as in the presence of MPGs consumers do not only buy the commodity under consideration, but in addition buy *an option* that if they are later informed of lower prices, they get the price difference back. Consumers value this option and this increases their reservation prices. Higher reservation prices, in turn, give firms the opportunity to raise their list prices, thereby increasing their profits. Thus, the key point of the paper is that the option value MPGs present impacts on the distribution of prices set by firms.

Another notable difference between our model and the existing literature is that we consider situations where consumers do not know in advance whether a firm that is visited has an MPG or not, i.e., MPGs are not pre-announced or advertised and consumers just encounter them when they arrive in a store. This setting where information about MPGs is revealed simultaneously with price information fits major consumer markets, such as electronics shops. In many of these shops, firms often put a label “minimum price guarantee” on their price labels, but not on their whole assortment. Moreover, at different points in time they have different products to which the MPG applies.³

We arrive at the following results. First, in our environment only two types of equilibria exist: one where firms do not set MPGs at all, and one where

³We do not want to argue that this setting where MPGs are not pre-announced applies to all markets and it is certainly an interesting question to investigate what market characteristics are more prone to pre-announced MPGs and where pre-announcements are not observed. We leave this as a question for future research.

firms set MPGs with a certain positive probability, which is strictly smaller than one. The latter equilibrium only exists when the level of communication among consumers is relatively high. Thus, importantly, an equilibrium where firms set MPGs for sure does not exist. This explains that in markets where MPGs are not announced, but only discovered when a customer arrives in the shop, firms randomize the products for which the MPG applies. This often happens in supermarkets and electronics stores. Second, the support of the equilibrium price distribution of a firm that provides MPG is always above the support of the distribution of a firm without MPG. This fact that firms setting MPGs have prices that are not below the prices of rivals firms without MPG is supported by empirical research (see Arbatskaya et al. (2004)). To understand the proper effect of MPGs empirically, our paper suggests, however, that one should not just compare prices in stores with and without MPGs, but instead one should also inquire whether the prices in stores without MPGs are shifted upwards when MPG *can be set* with some probability. Despite the fact that consumers can execute their MPG with some positive probability if they are informed of lower prices, consumers are strictly worse off. Moreover, the better consumers communicate, the higher the equilibrium prices and the higher the prices consumers expect to pay even taking the probability into account that consumers can execute the MPG. Finally, we consider the possibility of firms offering price-beating strategies and show that they are always dominated by price-matching policies. The reason is that in markets where MPGs are not pre-announced, *in equilibrium* MPGs only affect the reservation prices.

The structure of the paper is as follows. In the next section we present the setup of the model. Section three contains the equilibrium analysis and main comparative statics results. In section four we show that price-beating is never optimal. Section five briefly concludes.

2 Setup

Consider a market where two firms produce a homogenous good and have identical production costs, which we normalize to zero. Firms set prices and decide whether or not to provide minimum price guarantees (MPGs). By providing an MPG, a firm commits to compensate the difference between its price and the price of the competitor, if the customer who has bought the product from the firm provides evidence that the lower price exists.

Like in the model of Stahl (1989) there are two types of consumers. A fraction $\lambda \in (0, 1)$ of all consumers are “shoppers”, i.e. these consumers like shopping or have zero search costs for other reasons. We assume that these consumers know all prices in the market as well as whether some of the firms set minimum price guarantees. The remaining fraction $1 - \lambda$ of consumers is uninformed. These consumers engage in sequential search and get their first price quotation for free, but any subsequent price quotation comes at a search cost c . All consumers have identical valuation for the good denoted by v and $v > c$. We assume that v is non-binding in the model, i.e. it is sufficiently large

not to influence reservation prices. Whether a firm provides MPGs or not is revealed simultaneously with observing the price quotation of that firm. After the consumer has bought the good there is an exogenous probability $\mu \in (0, 1)$ that she observes (costlessly) the price of another firm. This information can come either from friends (as in Galeotti (2009)) or just accidentally because she noticed the price in the other store.

The timing in the model is as follows. First, firms simultaneously decide on their prices and minimum price guarantees. Firm i decides to set up minimum price guarantee with probability α^i , and then set prices with a probability distribution $F_0^i(p)$ if no minimum price guarantees set, and with $F_1^i(p)$ if it provides a MPG. Thus, the strategy of firm i is a tuple $\{\alpha^i, F_0^i(p), F_1^i(p)\}$. After firms made their decisions, consumers decide. Shoppers buy at the firm with the lowest price.⁴ After observing a first price, uninformed consumers have to decide whether to buy at that firm or to continue search. After all purchasing decisions have been made, customers have some probability of getting a price quotation of the firm, which they did not search. If this price is less than the purchase price, and the purchase was made in a firm providing an MPG, the customer costlessly claims the price difference, which is paid back by the firm.

We look for symmetric perfect Bayesian equilibria. In such an equilibrium firms choose the same probability of setting MPGs and choose prices with the same probability density function in case they do and do not set MPGs, i.e., we look for equilibria where $\{\alpha^1, F_0^1(p), F_1^1(p)\} = \{\alpha^2, F_0^2(p), F_1^2(p)\}$.

3 Analysis

We start our analysis by investigating the search behaviour of uninformed consumers. To this end, let us denote by $\{\underline{p}_j, \bar{p}_j\}$ the lower and upper bounds of $F_j(p)$, $j = 0, 1$ and $\underline{p} = \min\{\underline{p}_0, \underline{p}_1\}$. Let $F(p) = (1 - \alpha)F_0(p) + \alpha F_1(p)$ be the weighted average of the two equilibrium price distributions. Then the optimal search behaviour is defined by two reservation prices: one for firms with and another for firms without an MPG.

Lemma 3.1. *Uninformed consumers accept all prices at or below r_0 at a firm that does not provide MPG, and continue search otherwise; they accept all the prices at or below r_1 at a firm with MPG, and continue to search otherwise, where $\{r_0, r_1\}$ are defined by*

$$\begin{aligned} \int_{\underline{p}}^{r_0} F(p) dp &= c \\ \int_{\underline{p}}^{r_1} F(p) dp &= \frac{c}{1 - \mu} \end{aligned} \tag{1}$$

⁴In principle, if one of the firms charges the price lower than its competitor, while the competitor sets up the minimum price guarantees, shopper should be indifferent between the firms. We take one of possible models of their behaviour. One can think that there are infinitely small costs ϵ of claiming MPG, so shoppers prefer just to buy at the lowest price.

Proof. After observing price r_0 at a firm without minimum price guarantees, a consumer has to be indifferent between buying now and continuing to search. If the consumer continues to search, she proceeds to the next firm. The next firm does not set MPG with probability $1 - \alpha$, and in this case the consumer can choose the smallest price of r_0 and a random price p that is distributed according to F_0 . Similarly, for when she continues to search and happens to visit a store with MPG, which occurs with probability α . Therefore

$$\begin{aligned} r_0 &= c + (1 - \alpha) (F_0(r_0) \mathbb{E}_0(p|p < r_0) + (1 - F_0(r_0))r_0) \\ &+ \alpha (F_1(r_0) \mathbb{E}_1(p|p < r_0) + (1 - F_1(r_0))r_0) \end{aligned}$$

using integration by parts, this expression can be simplified to the usual rule determining the reservation prices.

$$\int_{\underline{p}}^{r_0} F(p) dp = c$$

Now consider the case when the customer found herself at a shop that provides MPG. In this case if the customer accepts the price there is a probability μ that later she observes another price, which is either from a no-MPG store (with probability $1 - \alpha$) or from a MPG store (with probability α). If she decides to continue searching, the situation is similar to the case described above. Therefore, the reservation price is defined by

$$\begin{aligned} (1 - \mu)r_1 &+ \mu[(1 - \alpha) (F_0(r_1) \mathbb{E}_0(p|p < r_1) + (1 - F_0(r_1))r_1) \\ &+ \alpha (F_1(r_1) \mathbb{E}_1(p|p < r_1) + (1 - F_1(r_1))r_1)] = \\ &= c + (1 - \alpha) (F_0(r_1) \mathbb{E}_0(p|p < r_1) + (1 - F_0(r_1))r_1) \\ &+ \alpha (F_1(r_1) \mathbb{E}_1(p|p < r_1) + (1 - F_1(r_1))r_1) \end{aligned}$$

which simplifies to

$$\int_{\underline{p}}^{r_1} F(p) dp = \frac{c}{1 - \mu}$$

□

It immediately follows from the lemma that $r_1 > r_0$, i.e., a consumer is willing to buy at a higher price if the firm happens to provide an MPG. This is quite natural as the consumer has a probability to receive a pay-back in case a MPG is provided. One can clearly see this happening for values of μ close to one. Indeed, if μ is close to one, then the customer visiting a firm with a MPG clause almost surely pays the minimum of the two prices in the market. If she decides to proceed to search then she pays c and again buys at the minimum of the two prices that are set. Thus, for high values of μ a consumer prefers to stop searching in the MPG store, almost independently of the price it observes there.

Now we turn to the equilibrium pricing behaviour of firms. It is a standard result in the consumer search literature that both $F_0(p)$, $F_1(p)$ are atomless and

that $\bar{p}_0 = r_0$, $\bar{p}_1 = r_1$. To provide a full characterization of equilibrium, we first show that certain types of equilibria cannot exist.

Proposition 3.2. *There is no symmetric equilibria with $0 < \alpha < 1$ and $r_0 > \underline{p}_1$.*

Proof. First, consider the profits of a firm which sets no minimum price guarantees. these profits are given by

$$\pi_0 = \lambda(1 - F(p))p + \frac{1 - \lambda}{2}p. \quad (2)$$

On the other hand, profits of a firm that provides MPG are equal to

$$\pi_1 = \lambda(1 - F(p))p + \frac{1 - \lambda}{2}((1 - \mu)p + \mu F(p)\mathbb{E}(p'|p' < p) + \mu(1 - F(p))p), \quad (3)$$

where the expectation is taken with respect to $F(p)$ and $p \leq r_1$. It is quite clear that equations (2) and (3) determine two different functional forms for $F(p)$. However, for the firm to be indifferent between setting and not setting a MPG it has to be the case that $\pi_0(p) = \pi_1(p)$ for all prices where the support of two distributions overlap. But this cannot be the case in more than one point, which together with that fact that $r_1 > r_0$ completes the proof. \square

Thus, if there is a positive probability that in equilibrium one firm provides MPGs, while the other does not, then it has to be the case that the price distributions are not overlapping. Given this result, we have three possible candidate equilibria: (i) $\alpha = 0$, (ii) $\alpha = 1$ and (iii) $0 < \alpha < 1$ but then $r_0 \leq \underline{p}_1$, i.e., a firm that does not provide an MPG will charge lower prices for sure than a firm with MPG.

We next argue that an equilibrium where both firms provide MPGs cannot exist either. In this, and the next propositions, it is important to realize that reservation prices are defined with respect to corresponding *equilibrium* price distributions. For example, if in an equilibrium both firms set MPG the consumer strategy is still represented by two reservation prices (r_0, r_1) , both of them are defined by (1) using $F(p) = F_1(p)$. A similar point holds true for an equilibrium candidate where none of the firms sets MPG.

Proposition 3.3. *There is no equilibrium where both firms play $\alpha = 1$.*

Proof. If one firm chooses $\alpha = 1$ then the competitor has a profitable deviation by choosing $\alpha = 0$ and price r_0 which (as $F(p) = F_1(p)$ in this case) is defined by $\int_{\underline{p}_1}^{r_0} F_1(p)dp = c$. Indeed, since it has to be the case that $\underline{p}_1 < r_0 < r_1$, r_0 lies in the support of $F_1(p)$ and we get

$$\begin{aligned} \pi(r_0) &= \lambda(1 - F(r_0))r_0 + \frac{1 - \lambda}{2}r_0 > \\ &> \lambda(1 - F(r_0))r_0 + \frac{1 - \lambda}{2}((1 - \mu)r_0 + \mu\mathbb{E}(\min(p, r_0))) = \pi_1. \end{aligned}$$

Therefore, there is a profitable deviation. \square

The idea behind this proposition is basically as follows. If a firm deviates from the proposed equilibrium where both firms provide MPG and simply sets the same price (in the lower end of the equilibrium distribution), but abandons the MPG, then the firm gets the same expected number of customers as with MPG, but the expected price paid by non-shoppers is higher since these consumers cannot exercise MPG anymore.

We now examine and characterize the remaining two candidate equilibria sequentially.

Proposition 3.4. *For all values of parameters there is an equilibrium where both firms choose $\alpha = 0$. The equilibrium price distribution in this case is*

$$F_0(p) = 1 - \frac{1 - \lambda}{2\lambda} \frac{r_0 - p}{p} \quad (4)$$

Proof. Since $F(p) = F_0(p)$ we have

$$\begin{aligned} \int_{\underline{p}}^{r_0} F_0(p) dp &= c \\ \int_{\underline{p}}^{r_1} F_0(p) dp &= \frac{c}{1 - \mu}. \end{aligned}$$

In equilibrium each firm gets a profit of $\pi_0 = \frac{1-\lambda}{2} r_0$. Assume, one firm deviates and provides MPGs. Then the highest possible profit that can be obtained is by charging $p = r_1$. Indeed, it is clear that a firm only benefits from the deviation if $p > r_0$ (otherwise it gets the same number of customers, but might experience a loss from searchers who can exercise MPG), but then the shoppers would not buy from this firm anyway, so the firm has to extract maximum profits from shoppers, which is attained by charging $p = r_1$. Then

$$\pi_1 = \frac{1 - \lambda}{2} ((1 - \mu)r_1 + \mu \mathbb{E}(p | p < r_1)) = \frac{1 - \lambda}{2} ((1 - \mu)r_1 + \mu(r_0 - c))$$

so that

$$\pi_1 > \pi_0 \Leftrightarrow r_1 - r_0 > \frac{\mu c}{1 - \mu}$$

But we have

$$\begin{aligned} r_1 - r_0 &= \int_{r_0}^{r_1} 1 dp = \int_{r_0}^{r_1} F_0(p) dp = \int_{\underline{p}}^{r_1} F_0(p) dp - \int_{\underline{p}}^{r_0} F_0(p) dp = \\ &= \frac{c}{1 - \mu} - c = \frac{\mu c}{1 - \mu} \end{aligned}$$

Thus, the best possible deviation gives the same payoff and a firm cannot strictly benefit from deviating. \square

Finally, we focus on the intermediate case where firms do provide MPGs with some probability. The following proposition establishes existence of an equilibrium in mixed strategies.

Proposition 3.5. *An equilibrium with $\alpha \in (0, 1)$ exists if and only if*

$$1 > \mu > \frac{4\lambda^2}{(1-\lambda)^2 \ln \frac{1-\lambda}{1+\lambda} + 2\lambda(1+\lambda)} > \frac{2}{3} \quad (5)$$

Proof. See Appendix. \square

This result might seem to be a bit counterintuitive: firms set up (with some probability) MPGs only if there is sufficient probability that customers would exercise it. The explanation of course is that if μ is sufficiently high, consumers would accept higher prices at the store with MPG which more than offsets the adverse effect of exercising MPG on firms profits.

The following two Figures represent the relationship between the equilibrium probability of observing minimum price guarantees and the parameters of the model. Equation (5) shows that the probability of observing other prices should be relatively large. Figure 1 depicts the relation between the equilibrium probability of firms offering MPGs and the probability with which consumers observe another price quotation. The figure shows this relationship is positive: α is increasing with μ . Though high values of μ imply that *ex post* most of the consumers are informed ones, uninformed consumers are willing to buy at higher prices for higher μ . If μ is close to one, customer's are willing to accept virtually any price lower than v . Therefore firms are more likely to set MPGs and sell at higher prices when μ is large. Not surprisingly, Figure 2 shows that the greater the fraction of shoppers λ the lower the probability with which firms set MPG.

Figure 1: Equilibrium probability of MPG as a function of μ at $\lambda = 0.2$.

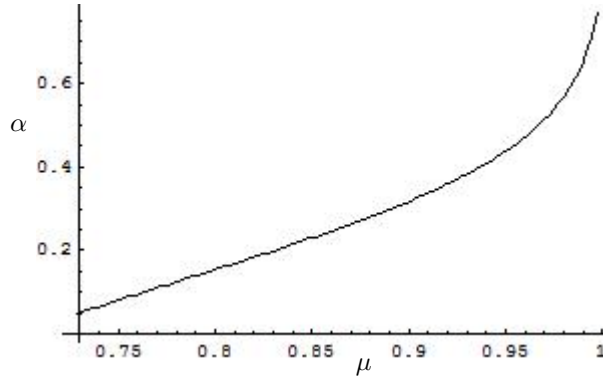
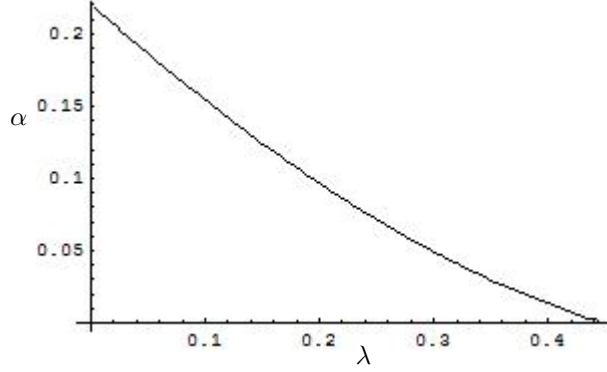


Figure 2: Equilibrium probability of MPG as a function of λ at $\mu = 0.8$.



Now we proceed with the comparative statics analysis. The following proposition compares expected prices paid by consumers in the equilibrium with MPG and in the equilibrium without it.

Proposition 3.6. *Expected profits for firms in the equilibrium where MPGs are offered with positive probability are higher than the expected profits in the equilibrium without MPGs. As a consequence, in the equilibrium where MPGs are offered consumers pay higher expected prices (after a possible execution of MPG) than in the equilibrium without MPGs.*

Proof. In fact, the equilibrium without MPG described by the same formulas as the equilibrium with MPG when α is set to be zero. The level of equilibrium profits for the equilibrium with MPG is

$$\pi(\alpha) = \frac{\lambda(1 - \lambda + 2\alpha\lambda)}{2(1 - \alpha)\lambda + (1 - \lambda + 2\alpha\lambda) \ln \left(\frac{1 - \lambda + 2\alpha\lambda}{1 + \lambda} \right)} c$$

Then

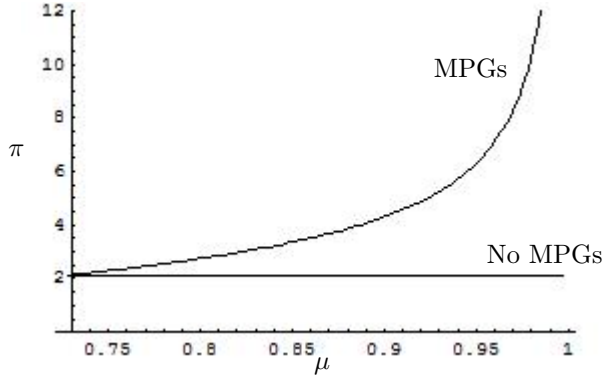
$$\frac{\partial \pi}{\partial \alpha} = \frac{4(1 - \alpha)\lambda^3}{\left(2(1 - \alpha)\lambda + (1 - \lambda + 2\alpha\lambda) \ln \left(\frac{1 - \lambda + 2\alpha\lambda}{1 + \lambda} \right) \right)^2} c > 0$$

□

Proposition 3.7 shows the “anticompetitive” effect of a MPG in a search environment in the sense that in the equilibrium with MPGs the expected price is higher than in the equilibrium where MPGs are not offered for sure. The source of the anticompetitive effect is, however, different from that so far studied in the literature. It is not the case here that there is some type of collusive behaviour

between the firms where MPGs play the role of a monitoring device. In our case the result is fully driven by consumer behaviour, namely by the willingness of consumers to accept higher prices when firms do offer MPG. Another interesting observation is that the higher expected price paid in the equilibrium with MPG comes from two sources: (i) A firm charging an MPG can set a higher price on average because of the higher reservation price of consumers at firms with an MPG and (ii) other firms without MPG react to these higher prices by setting higher prices themselves. Thus, also a firm that effectively does not charge MPGs has a higher price paid in the equilibrium where MPGs are charged with some positive probability compared to the equilibrium without MPGs. Figure 3 shows how a firm's expected profits depend on μ in the equilibrium where MPGs are offered with some probability and the equilibrium profits without MPGs. In the latter case, profits are, of course, a constant, whereas they are exponentially increasing in μ whenever this equilibrium exists.

Figure 3: Expected profits as a function of μ at $\lambda = 0.2$.



4 Price-beating and free-lunch strategies

We next turn to the question whether firms will ever choose to provide price beating guarantees (PBG), if they do not announce (advertise) this policy in advance before consumers search for prices. To study this question, assume firms not only guarantee the purchase at the minimum price in the market if a lower price has been observed, but to compensate the customer even further by informing the customer that if she provides evidence of a lower price, say p' , the effective purchase price will be $\beta p'$, where $\beta \leq 1$. The probability with which a price-beating strategy is chosen is (again) denoted by the probability α .

Even without a characterisation of the optimal search rule, we can argue that it is never optimal for a firm to offer a price beating strategies if it does not

announce this policy in advance.⁵ In the following two propositions we assume that prices charged are less or equal to corresponding reservation prices.

Proposition 4.1. *It is never optimal for a firm to offer a price beating policy with $\beta < 1$.*

Proof. Consider the profit function of a firm in an equilibrium where it provides a price-beating policy β :

$$\begin{aligned} \pi &= \lambda((1 - \alpha)(1 - F(p))p + F(p)\mathbb{E}(\beta p' | p' < p)) \\ &+ \frac{1-\lambda}{2}((1 - \mu)p + \mu(1 - F(p))p + \mu F(p)\mathbb{E}(\beta p' | p' < p)) \end{aligned} \quad (6)$$

The first term of this formula represent the profit the firm gets from informed customers. These consumers buy when this firm either has the highest price as in that case they buy in order to exercise the price beating guarantee, or when the other firm has a higher price and does not offer a price beating policy. The last term represents the profit from uninformed customers. These consumers effectively buy at price p either because they are not informed about another price, or because the other firm charges a higher price. If the other firm charges a lower price, these consumers effectively pay a fraction of this lower price if they are informed about it. If the firm deviates and charges a higher β , it receives a higher profit as $\frac{\partial}{\partial \beta} \mathbb{E}(\beta p' | p' < p) > 0$ and the equilibrium price distribution is unaffected by the deviation. Thus, we have $\frac{\partial \pi}{\partial \beta} > 0$ and therefore it is optimal for a firm to set $\beta = 1$. \square

This result shows that in markets where PBGs are not announced to consumers in advance so that consumers only are aware of these guarantees once they are in the shop and see the prices offered, price-matching is always preferred to price-beating. This comes from the fact that price-beating in these markets only affects the price the firm receives from those consumers who know the other firm has a lower price, but it does not affect the number of consumers. Therefore, for each individual firm it is better to choose $\beta = 1$, though if both firms were to stick to some $\beta < 1$ it would result in higher profits per firm.

Instead of offering price-beating strategies companies could compensate the price difference (match the price) and offer a “free lunch” on top of that. Thus, instead of offering $\beta p'$ the firm offers $p' - x$, where x is the value of the “lunch”. The analysis and the results are very similar, as the following proposition shows.

Proposition 4.2. *It is never optimal for a firm to offer a price beating policy with $x > 0$.*

Proof. Consider the profit function of a firm in an equilibrium where it offers:

$$\begin{aligned} \pi &= \lambda((1 - \alpha)(1 - F(p))p + F(p)\mathbb{E}(p' - x | p' < p)) \\ &+ \frac{1-\lambda}{2}((1 - \mu)p + \mu(1 - F(p))p + \mu F(p)\mathbb{E}(p' - x | p' < p)) \end{aligned} \quad (7)$$

⁵<http://www.besparingsmeter.nl:80/2009/06/17/winkelen/sla-je-slag-bij-ah/> reports cases where students bring cases with beer out of the supermarket Albert Heijn. After that Albert Heijn stopped this policy.

$\frac{\partial \pi}{\partial x} < 0$ because $\frac{\partial}{\partial \beta} \mathbb{E}(p' - x | p' < p) < 0$. Therefore it is optimal for the firm to choose x as low as possible, therefore $x = 0$. \square

5 Conclusions

This paper analyzed the effect of firms offering price matching and price beating strategies in a consumer search model where reservation prices are endogenously determined. We restrict the analysis to markets where consumers are uninformed about whether or not firms offer minimum price guarantees before they come to the shop. We show that the effects of price matching and price beating are very different. Price matching can be observed in equilibrium, but only as an outcome of an equilibrium where firms randomize the decision to set minimum price guarantees. This may explain why multi-product firms (such as supermarkets and electronics shops) offer these policies over an ever changing group of products (if they offer them at all). We show that in the equilibrium where firms offer minimum price guarantees with strictly positive probability, the expected prices consumers pay are higher than in the equilibrium where no firm sets minimum price guarantees. The main reason for this result is that consumers' reservation prices increase considerably as they factor in the probability that they will be informed about lower prices later (and get their money back) and therefore they are more eager to buy now even if the price is relatively high. Importantly, even a shop that does not use minimum price guarantees sets higher prices on average. This basically follows as MPGs soften the competition.

Price beating strategies are different and it is never optimal to set them. The main reason is that firms do not gain additional consumers by setting such policies and only risk to get lower expected prices in case the competitor has a lower price.

Appendix: Proof of Proposition 3.6

To prove the proposition we explicitly construct an equilibrium and then show, that there is such a value of α that all the equilibrium conditions are satisfied.

Equilibrium price distribution support contains two parts: $[\underline{p}, r_0] \cup [\underline{p}_1, r_1]$.

The lower part of the support. The lower part of the support is defined by three equations:

$$\pi = \lambda(1 - F(p))p + \frac{1 - \lambda}{2}p$$

$$F(\underline{p}) = 0$$

$$F(r_0) = 1 - \alpha$$

These three equations allow to write down everything as a function of (r_0, α) .

Indeed,

$$\begin{aligned}\pi(r_0, \alpha) &= \frac{2\alpha\lambda + 1 - \lambda}{2} r_0 \\ \underline{p}(r_0, \alpha) &= \frac{2\alpha\lambda + 1 - \lambda}{1 + \lambda} r_0 \\ F(p; r_0, \alpha) &= \frac{(1 + \lambda)p - (1 - \lambda + 2\alpha\lambda)r_0}{2\lambda p}\end{aligned}\tag{8}$$

Which using optimal search rule

$$\int_{\underline{p}}^{r_0} F(p) dp = c$$

gives an expression for r_0 :

$$r_0 = \frac{2\lambda c}{2(1 - \alpha)\lambda + (1 - \lambda + 2\alpha\lambda) \ln \left(\frac{1 - \lambda + 2\alpha\lambda}{1 + \lambda} \right)}\tag{9}$$

Thus, if the value of α is known the probability distribution on the lower part of the support is fully described.

The upper part of the support. Now let's consider the upper part.

The profit function is defined by:

$$\pi = \lambda(1 - F(p))p + \frac{1 - \lambda}{2} \left(p - \mu \int_{\underline{p}}^p F(q) dq \right)\tag{10}$$

Since $\int_{\underline{p}}^{r_1} F(p) dp = \frac{c}{1 - \mu}$ we get

$$\pi = \frac{1 - \lambda}{2} \left(r_1 - \frac{\mu c}{1 - \mu} \right)$$

Same way since $\int_{\underline{p}}^{\underline{p}_1} F(p) dp = c + (1 - \alpha)(\underline{p}_1 - r_0)$ we get

$$\pi = \alpha\lambda\underline{p}_1 + \frac{1 - \lambda}{2} \left(\underline{p} - \mu(c + (1 - \alpha)(\underline{p}_1 - r_0)) \right)$$

Now, using equations (8) and (9) we can get expressions for r_1 and \underline{p}_1 as functions of α .

$$r_1 = \frac{2\lambda(1 - \lambda(1 - (2 - \mu)\alpha) + \alpha\mu) + (1 - \lambda)(1 - \lambda + 2\alpha\lambda)\mu \ln \frac{1 - \lambda + 2\alpha\lambda}{1 + \lambda}}{(1 - \lambda)(1 - \mu) \left(2(1 - \alpha)\lambda + (1 - \lambda + 2\alpha\lambda) \ln \frac{1 - \lambda + 2\alpha\lambda}{1 + \lambda} \right)} c$$

$$\underline{p}_1 = \frac{(1 - \lambda + 2\alpha\lambda) \left(2\lambda + (1 - \lambda)\mu \ln \frac{1 - \lambda + 2\alpha\lambda}{1 + \lambda} \right)}{(1 - \lambda(1 - (2 - \mu)\alpha - \mu) - \mu(1 - \alpha)) \left(2(1 - \alpha)\lambda + (1 - \lambda + 2\alpha\lambda) \ln \frac{1 - \lambda + 2\alpha\lambda}{1 + \lambda} \right)} c$$

Determination of α . To determine the value of α we use the following approach. We solve for the probability distribution on the upper part of the support using differential equation (10). The solution requires determination of the constant, say Q using boundary condition. We have two of them: $F(\underline{p}_1) = 1 - \alpha$ and $F(r_1) = 1$, which gives us two values of the constant Q_1 and Q_2 . Since the solution must satisfy both boundary conditions we get have to get $Q_1 = Q_2$ which gives us the equation on α . Note, that we do not calculate the optimal search integral here, since it is already incorporated in determination of r_1 ⁶.

We start with the following differential equation:

$$Ay(x) + Bxy'(x) + Cx + D = 0 \quad (11)$$

The solution of this equation is

$$y(x) = Qx^{A/B} - \frac{Cx}{A+B} - \frac{D}{A}$$

Now, if we compare (11) with (10) we can spot that it is the same equation with $x = p$, $y(x) = \int_{\underline{p}}^p F(p)dp$, $y'(x) = F(p)$, $A = -\frac{1-\lambda}{2}\mu$, $B = -\lambda$, $C = \frac{1+\lambda}{2}$, $D = \pi$.

Thus, the equilibrium price distribution is defined by

$$F(p) = \frac{1+\lambda}{2\lambda+(1-\lambda)\mu} - Q \frac{(1-\lambda)\mu}{2\lambda} p^{-\frac{2\lambda+(1-\lambda)\mu}{2\lambda}}$$

where Q is determined by initial conditions. $F(r_1) = 1$ and $F(\underline{p}_1) = 1 - \alpha$ give two values Q_1 and Q_2 which has to be equal.

$$Q_1 = -2\lambda(1-\lambda)\mu \left(1 - \frac{1+\lambda}{2\lambda+(1-\lambda)\mu} \right) \left(\frac{c(2\lambda(1-\lambda(1-\alpha(2-\mu)-\mu)-\mu(1-\alpha))-(1-\lambda)(1-(1-2\alpha)\lambda)\mu \ln \frac{1-\lambda+2\alpha\lambda}{1+\lambda})}{(1-\lambda)(1-\mu)(2(1-\alpha)\lambda+(1-(1-2\alpha)\lambda) \ln \frac{1-\lambda+2\alpha\lambda}{1+\lambda})} \right)^{1+\frac{(1-\lambda)\mu}{2\lambda}}$$

$$Q_2 = -2\lambda(1-\lambda)\mu \left(1 - \alpha - \frac{1+\lambda}{2\lambda+(1-\lambda)\mu} \right) \left(\frac{c(1-(1-2\alpha)\lambda)(2\lambda+(1-\lambda)\mu \ln \frac{1-\lambda+2\alpha\lambda}{1+\lambda})}{(1-\lambda(1-\alpha(2-\mu)-\mu)-\mu(1-\alpha))(2(1-\alpha)\lambda+(1-(1-2\alpha)\lambda) \ln \frac{1-\lambda+2\alpha\lambda}{1+\lambda})} \right)^{1+\frac{(1-\lambda)\mu}{2\lambda}}$$

Equation $Q_1 = Q_2$ can be reduced to:

⁶Another, may be more natural approach, is to use just one boundary condition and then explicitly calculate the search integral to get the equation for r_0 and α , as we did for the lower part of the support. However this approach results in more analytical complications, so we use the one presented in the text.

$$\left(\frac{(1-\lambda)(1-\mu)}{1-\lambda(1-\alpha(2-\mu)-\mu)-(1-\alpha)\mu} \right)^{\frac{(1-\lambda)\mu}{2\lambda}} = \left(\frac{2\lambda(1-\lambda+2\alpha\lambda-\alpha(1+\lambda)\mu) + (1-\lambda)(1-\lambda+2\alpha\lambda)\mu \ln \frac{1-\lambda+2\alpha\lambda}{1+\lambda}}{(1-\lambda+2\alpha\lambda) \left(2\lambda + (1-\lambda)\mu \ln \frac{1-\lambda+2\alpha\lambda}{1+\lambda} \right)} \right)^{\frac{2\lambda+(1-\lambda)\mu}{2\lambda}} \quad (12)$$

First, we evaluate (12) at $\alpha = 0$. It is easy to spot that both LHS and RHS takes values of 1 for all (λ, μ) . Second, we claim that the LHS of equation (12) evaluated at $\alpha = 1$ is greater than the RHS. Indeed, after canceling some terms the equation can be rewritten as $\left(\frac{1-\lambda}{1+\lambda} \right)^{\frac{(1-\lambda)\mu}{2\lambda}} = (1-\mu)$. Thus, the LHS is increasing in λ and as $\lambda \rightarrow 0$ it goes to e^μ which is greater than $(1-\mu)$. Finally, we examine the behaviour both of LHS and RHS around $\alpha = 0$. Obviously, if LHS decreases faster than the RHS, there must be an intersection point at $\alpha \in (0, 1)$. The derivative of the LHS with respect to α evaluated at $\alpha = 0$ equals to $-\frac{\mu(\lambda(2-\mu)+\mu)}{2\lambda(1-\mu)}$. The derivative of RHS evaluated at $\alpha = 0$ equals to $-\frac{(1+\lambda)(\lambda(2-\mu)+\mu)\mu}{(1-\lambda)(2\lambda+(1-\lambda)\mu \ln \frac{1-\lambda}{1+\lambda})}$. Solving

$$-\frac{\mu(\lambda(2-\mu)+\mu)}{2\lambda(1-\mu)} < -\frac{(1+\lambda)(\lambda(2-\mu)+\mu)\mu}{(1-\lambda) \left(2\lambda + (1-\lambda)\mu \ln \frac{1-\lambda}{1+\lambda} \right)}$$

gives $\mu \in (-\frac{2\lambda}{1-\lambda}, 0) \cup (\frac{4\lambda^2}{(1-\lambda)^2 \ln \frac{1-\lambda}{1+\lambda} + 2\lambda(1+\lambda)}, \infty)$. Given that μ is between 0 and 1 we get (5).

Now we show that $f(\lambda) \equiv \frac{4\lambda^2}{(1-\lambda)^2 \ln \frac{1-\lambda}{1+\lambda} + 2\lambda(1+\lambda)} > 2/3$. First, this expression is increasing in λ with $f(1) = 1$. Second, we take a limit $\lim_{\lambda \rightarrow 0} f(\lambda)$. By applying l'Hopital's rule twice we get:

$$\lim_{\lambda \rightarrow 0} f(\lambda) = \frac{8}{4 \frac{3+\lambda(3+\lambda)}{(1+\lambda)^2} + 2 \ln \frac{1-\lambda}{1+\lambda}} = \frac{2}{3}$$

To prove that (5) is also a necessary condition we show that if the derivative of LHS of (12) is greater than the derivative of the RHS at $\alpha = 0$ then the LHS is higher than the RHS for any other α . That implies that there is no such a value of α which can equate both sides of the equation. Note, that both LHS and RHS of (12) are smooth functions in α, λ, μ . Therefore, we can directly verify the result on a mesh for $(\alpha, \lambda, \mu) \in (0, 1)^3$. Numerical verification shows that (5) is indeed not only sufficient, but a necessary condition as well.

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